

Contents

Abstract	i
Kurzfassung (German abstract)	iii
1 Introduction	1
1.1 Synchronization of multi-agent systems	1
1.2 Synchronization problem and performance specifications	2
1.3 Literature review	4
1.4 Structure and contribution of this thesis	9
2 Preliminaries	13
2.1 Notations and definitions	13
2.2 Graph theory	16
2.3 Review of the LQR design method	20
2.4 Review of the root locus design method	21
2.5 Introduction to gossip algorithms	24
2.6 Demonstration example: Autonomous mobile robots	25
3 Optimal synchronization of multi-agent systems	29
3.1 Introduction to optimal synchronization	29
3.2 Agent model	31
3.3 Static networked controller	32
3.4 Model of the overall closed-loop system	33
3.5 Synchronization condition	34
3.6 LQR design problem for synchronization	35
3.7 LQ-Regulator for the synchronization of multi-agent systems	37
3.7.1 Decomposition of the design problem	37
3.7.2 Optimal synchronization of completely coupled agents	40

3.8	Approximate method for the design of synchronizing controllers for non-complete networks	43
3.9	Optimal synchronization in non-complete networks	47
3.9.1	LQ-like design problem for synchronization	47
3.9.2	Solution of the optimization problem	49
3.9.3	Algorithm for the design of optimal synchronizing controllers	51
3.10	Application example: Synchronization of a vehicle platoon	52
4	Output synchronization with guaranteed performance	59
4.1	Introduction and performance requirements	59
4.2	Agents with output disturbances	61
4.3	Dynamic networked controller	63
4.4	Model of the overall system and assumptions	64
4.5	Decomposition and synchronization condition	66
4.6	Design of dynamic networked controllers	69
4.6.1	Design idea	69
4.6.2	Transient behavior of the decoupled synchronization errors	71
4.6.3	Bound on the overall synchronization error	72
4.6.4	Controller design algorithm	76
4.7	Extensions	81
4.7.1	Synchronization of agents with non-identical dynamics	81
4.7.2	Synchronization in directed communication networks	83
4.8	Application example: Synchronization of a vehicle platoon	85
4.8.1	Vehicle model and control aim	85
4.8.2	Synchronization of vehicles in undirected networks	86
4.8.3	Synchronization of vehicles in directed networks	88
5	Gossip algorithms for the synchronization of multi-agent systems	93
5.1	Synchronization in point-to-point networks	93
5.2	Agent model and assumptions	95
5.3	Synchronizing controller design	96
5.3.1	Structure of the networked controller	96
5.3.2	Design of a synchronizing dead-beat controller	97
5.3.3	Behavior of the networked multi-agent system	99
5.4	Deterministic agent coupling	102
5.4.1	Control algorithm for repetitive synchronization of agent pairs	102

5.4.2	Synchronization condition	105
5.4.3	Design of the sampling time to guarantee performance	106
5.5	Random agent coupling	111
5.5.1	Control algorithm for random synchronization of agent pairs	111
5.5.2	Synchronization condition	114
5.5.3	Design of the sampling time to guarantee performance	115
5.6	Example: Synchronization of harmonic oscillators	119
5.6.1	Model of harmonic oscillators	119
5.6.2	Synchronization of harmonic oscillators	120
5.6.3	Synchronization of unstable oscillators	124
6	Conclusions	127
Bibliography		129
Appendices		141
A	Proofs of Chapter 3	143
A.1	Proof of Theorem 3.1	143
A.2	Proof of Lemma 3.3	144
A.3	Proof of Lemma 3.4	149
B	Proofs of Chapter 4	151
B.1	Proof of Theorem 4.1	151
B.2	Proof of Theorem 4.3	152
C	Proofs of Chapter 5	155
C.1	Proof of Theorem 5.1	155
C.2	Proof of Theorem 5.3	156
D	List of Symbols	159