Abstract

In the design of many modern control systems such as networked control systems (NCS) and real-time systems with shared computational resources, effects caused by sampling cannot simply be neglected. On the one hand, aperiodic sampling patterns occur for such systems, for example due to packet losses and delays in NCS or caused by the limited availability of shared computational resources for real-time systems. On the other hand, due to resource constraints that are caused, e.g., by bandwidth limitations for NCS and limited availability of shared computation hardware for real-time systems, it can be advantageous to proactively design aperiodic sampling patterns. Doing so has the potential to reduce the number of sampling instants needed and thus to save resources. To address the resulting need for methods to analyze and design control systems with aperiodic sampling, two recent streams of research focus on the topic. The first stream focuses on the analysis of the effects caused by aperiodic sampling patterns in feedback control with the goal to assure the proper functionality of control systems despite sampling effects. The second stream focuses on the development of event-based control strategies, which only release new samples when states or outputs of the system change significantly. Event-based control potentially allows saving resources required for sampling while still guaranteeing a certain performance for the control system. The most popular concepts in the context of event-based control are event-triggered control (ETC), where sampling instants are determined according to a state-dependent triggering rule that is continuously monitored, and self-triggered control (STC), where the next sampling instant is determined at each sampling instant using information available at this time.

With this thesis, we contribute to both research streams on aperiodic sampling with a specific focus on nonlinear control systems. A problem that typically arises in the first stream is to determine an upper bound on the sampling intervals, called maximum sampling interval (MSI), such that stability or performance can be guaranteed for any arbitrary sampling pattern that satisfies this bound. However, the analysis solely based on the MSI, which is typically carried out in the literature, only considers the worst-case scenario for the system and neglects other information, such as average bounds on the sampling intervals. In this thesis, we demonstrate that considering additional average constraints on the sampling intervals may have significant benefits in the MSI analysis. In particular, we show qualitatively that in a standard nonlinear control setting often considered in the literature, arbitrarily large finite values for the MSI can be obtained if the average sampling interval is sufficiently small. We thus highlight that average constraints on sampling intervals can be even more important than the MSI when analyzing systems with aperiodic sampling. On a technical level, the result is enabled by a novel hybrid Lyapunov function construction based on a hybrid clock variable.

Regarding the second research stream, i.e., the analysis and design of event-based control strategies, this thesis contains the following contributions.

As first contribution, we propose a framework for ETC of nonlinear systems based on $\mathcal{L}_p$ norms. The framework is based on interpreting the considered nonlinear system with event-triggered sampling as a feedback interconnection of two subsystems. The first subsystem describes the behavior of plant and controller states and the second subsystem the evolution of the sampling-induced error. Based on this setup, we derive a class of triggering rules that trigger sampling whenever the $\mathcal{L}_p$ gain from the chosen output to the sampling-induced error is below a certain threshold. The triggering rules thus guarantee a specific $\mathcal{L}_p$ gain for the subsystem that describes the sampling-induced error. Guarantees for a finite $\mathcal{L}_p$ gain of the closed-loop system are derived based on small-gain arguments. The framework covers various existing triggering rules from the literature and provides novel $\mathcal{L}_p$ stability results for such triggering rules. Moreover, since it is based on technical concepts that differ from those in the literature, it provides novel interpretations for such existing triggering rules like that they can be interpreted as $\mathcal{L}_p$ gain conditions for the subsystem that describes the sampling-induced error. It further leads to novel classes of triggering rules based on $\mathcal{L}_p$ norms and offers flexibility regarding the resulting closed-loop properties.

As second contribution, we present a systematic approach to design ETC triggering rules for linear systems based on integral quadratic constraints (IQCs). We interpret the system with ETC again as a feedback interconnection, but this time use the ETC triggering rule to ensure that the subsystem that describes the sampling-induced error satisfies a designed IQC. This allows us to leverage the IQC framework to provide a systematic procedure for the design of triggering rules based on linear matrix inequalities and to obtain guarantees for stability and performance. Whilst the proposed framework for ETC based on $\mathcal{L}_p$ norms addresses a broad class of systems and triggering rules, the proposed approach based on IQCs focuses on specific triggering rules and linear systems. This restriction in turn allows a systematic and numerically easy implementable approach for the design and analysis of triggering rules.

The third contribution regarding event-based control is that we propose an approach for self-triggered control for nonlinear systems. Only few STC approaches in the literature deal with nonlinear systems. In particular, there is a lack of approaches that can be used for perturbed nonlinear systems without known disturbance bound. Moreover, existing STC approaches typically only take the current system state into account but not the past system behavior, which also depends on past sampling instants. Both aspects are addressed by the proposed dynamic STC approach, which leverages a dynamic variable and hybrid Lyapunov functions that include hybrid clock variables. We provide guarantees for input-to-state stability and, if the disturbance is bounded, robust asymptotic stability of a robust invariant set.

Chapter 1

Introduction

In classical digital control setups, feedback controllers are implemented using periodic sampling with a fixed sampling period [10], [38]. In such setups, it is typically assumed that the sampling period is fixed as soon as specific digital hardware has been selected. Hereby, a trade-off between control performance and hardware cost is aimed for, often using rules of thumb or simulations [36], [37]. In the classical literature like [10], [38], alternatives for sampling with a fixed sampling period are often not thematized. In recent decades, however, it has turned out that in many situations it can instead be advantageous to sample in an aperiodic fashion. Sampling intervals can either be assigned to the system externally or be determined in an event-based fashion depending on the state of the system [49], [58], [145].

Aperiodic sampling is particularly important in the area of networked control systems (NCS). Such systems, in which communication networks replace dedicated point-to-point connections in the feedback loop, have gained a lot of popularity in the past years. Due to advances in network technology, there are now a large number of practical applications for NCS, e.g. summarized in the surveys [17], [57]. In NCS, network effects occur that have the potential to pose significant restrictions on the systems operation. Bandwidth constraints in the feedback-loop can limit the rate at which information can be sent [57]. Congestion in overloaded networks can result in delays and packet loss, that may deteriorate control performance [129]. Moreover, in shared and distributed network setups, it can occur that only a subset of the sensors can communicate at the same time [130]. It is therefore essential in the design of NCS to analyze aperiodic time-varying sampling patterns generated by network effects [58]. In addition, the network induced challenges can be mitigated by reducing the network load. To this end, it may be beneficial to use event-based sampling in NCS to avoid unnecessary sampling instants, which has the potential to reduce the required amount of transmissions and the bandwidth requirements [20], [104].

Similar effects also occur in real-time systems with shared computational resources. Here overload situations may, e.g., lead to aperiodic sampling patterns that can be dealt with by a suitable analysis [79]. Moreover, event-based sampling may help to save resources by releasing CPU time for other tasks [5], [8], [120], [128]. For systems with resource limitations like in satellite control, it may further be beneficial to reduce the number of actuation updates which can also be achieved by event-based sampling [96]. Finally, aperiodic sampling is natural for certain system classes like systems with impulsive inputs [9] or in the control of epidemics [30].

Reflecting the need for aperiodic sampling in various classes of control systems, there has been an increasing interest in developing control theory for systems with aperiodic sampling in the recent decades (documented, e.g., in the surveys [49], [58], [145]). This includes developing approaches for the design of systems with aperiodic sampling, understanding the underlying effects and to give rigorous control theoretic guarantees. There are two main research streams for aperiodic sampling that differ in the way how sampling instants are determined. The first stream focuses on systems with time-varying sampling intervals that are prescribed to the system externally independent from the state of the control system (surveyed, e.g., in [58]). The focus of the second stream are systems with event-based sampling mechanisms, for which sampling instants are determined based on triggering rules that take the state of the control system into account (surveyed, e.g., in [49]).

The goal of the first research stream is to provide techniques to analyze the effect that aperiodic sampling with time-varying sampling intervals has on control systems. Typical questions that arise in the first research stream include the (offline) choice of sampling intervals, the evaluation of required computational and energy resources and the synthesis of controllers with suitable robustness properties. These questions often boil down to the problem of determining the maximum sampling interval (MSI). The MSI is a bound for the length of the sampling interval so that stability or performance can be guaranteed for arbitrary sampling patterns with sampling intervals that do not exceed the MSI [58]. Different modeling and analysis techniques are proposed in the literature to determine the MSI, including time-delay systems [40], hybrid systems [93] and input-output models [87]. Whilst a large number of different approaches to calculate MSIs already exists in recent literature (summarized e.g. in [58], [145]), it is in general not clear which approach is ideal for which setup and research on the MSI problem is still going on. Existing approaches are often conservative when it comes to nonlinear systems, so that improvements of existing analysis techniques are desirable. It has further been demonstrated that using additional information on the sampling intervals as, e.g., an average constraint on the length of the sampling intervals, can significantly reduce the conservatism of the MSI analysis [54]. With this thesis, we contribute to the first research stream by extending this insight even further, and illustrate that average constraints on the sampling interval may be even more important for the analysis of control theoretic properties than the MSI. In particular, we show qualitatively that in a standard nonlinear control setting adopted from [54], [93], arbitrarily large finite values for the MSI can be obtained if the average sampling interval is sufficiently small. We thus highlight that average constraints on sampling intervals can be even more important than the MSI when analyzing systems with aperiodic sampling.

The goal of the second research stream, which focuses on event-based sampling, is to proactively design the sampling pattern by determining sampling instants at the runtime of the control system. Early works on event-based sampling have shown that for simple setups, event-based sampling with a simple threshold-based triggering condition provably outperforms periodic control with a fixed sampling period. In particular, the variance for first order stochastic systems can be reduced significantly at the same average sampling rate [9]. Since then, various approaches for event-based sampling have been developed. The approaches can be classified into two main categories. The most prevalent is event-triggered control (ETC), a reactive approach, where a triggering rule is monitored continuously and a sampling instant is triggered as soon as the triggering rule is violated. A more proactive alternative is self-triggered control (STC), where the next sampling instant is precomputed at each sampling instant based on available state information and on predictions [49].

We present in this thesis the following contributions to the second research steam, which are summarized in more detail in Section 1.2. First, we introduce a framework for ETC of nonlinear systems based on measured $\mathcal{L}_p$ norms and small-gain arguments. The framework covers various existing triggering rules from the literature and provides novel interpretations for such triggering rules like that they can be interpreted as $\mathcal{L}_p$ gain conditions. It further leads to novel classes of triggering rules based on $\mathcal{L}_p$ norms and offers flexibility regarding the resulting closed-loop properties.

As second contribution, we propose a systematic approach to design ETC triggering rules for linear systems based on integral quadratic constraints (IQCs). Leveraging tools from robust control theory, we derive conditions for stability and performance in terms of linear matrix inequalities (LMIs). Whilst the proposed framework based on $\mathcal{L}_p$ norms is kept general and addresses a general system class and various different triggering rules, the approach to design triggering rules based on IQCs considers linear systems and specific triggering rules. In turn, it allows for a systematic and numerically easy implementable approach for the design and analysis of triggering rules.

As third contribution, we propose a dynamic approach for STC of nonlinear systems. The dynamic STC approach leverages a dynamic variable and hybrid Lyapunov functions, which include hybrid clock variables. For the proposed dynamic STC approach, we provide guarantees for input-to-state stability and, if the disturbance is bounded, robust asymptotic stability of a robust invariant set.

In the next section, we provide an overview over related literature for the thesis, before summarizing our contributions in more detail in Section 1.2.

1.1 Related work

In this section, we provide an overview over related literature for both research streams with a particular focus on literature relevant in context of the results that we present in this thesis.

1.1.1 Aperiodic sampling with time varying sampling intervals and the MSI problem

In this subsection, we review the literature on aperiodic sampling with time varying sampling intervals with a particular focus on literature concerning the MSI problem. A historical basis for the analysis of aperiodic sampling is provided by methods for analyzing sampled-data systems with fixed sampling period, which were already studied in the 1950s [70]. A large variety of methods for the controller design for systems with constant sampling period based on the discretization of the considered system is available (see, e.g., [10], [23]). However, even for setups with fixed sampling period, there are two major drawbacks of such discrete-time designs. First, the inter-sampling behavior is usually not taken into account by such designs [22] and second it is in general difficult to find exact discretizations, particularly when considering nonlinear systems [94]. An alternative that avoids the above issues and that can also easily be adopted for aperiodic sampling is controller emulation. Controller emulation is a two step procedure. In a first step, a controller for the continuous-time system is designed, ignoring sampling effects. Then, the effects of sampling on the closed-loop system are analyzed in a second step [64], [65]. Early works on emulation show qualitatively that for sufficiently fast sampling, stability properties of the continuous-time system are preserved. Such results exist for systems with constant sampling period [55], [123] and for systems with time varying sampling period [60], [97]. For the implementation of sampled-data systems, however, more quantitative results that provide specific MSI bounds are required. To that end, various approaches have been proposed in the literature that can be classified into different frameworks. Three frameworks have proven to be particularly useful to classify the approaches.

In the time-delay framework, the sampling effects are modeled as fast time-varying delays. Then analysis tools that were originally developed for systems with delays are leveraged. In [40], a Lyapunov-Krasovskii functional is used for the MSI analysis. Using over-approximation techniques, LMI conditions for the MSI problem are derived for linear systems. Since the original approach from [40] bears some conservatism, various improvements with more sophisticated Lyapunov-Krasovskii functionals have been proposed, see, e.g., [39], [98], [113]. The approach can also be extended to controller synthesis [40], [71]. Moreover, extensions to nonlinear systems [62], [80] and to discrete-time systems [115] exist. Similar analysis techniques are also used for a looped-functional approach in [114], [144].

Closely related to the time-delay framework is the input-output framework, in which sampling effects are considered as a perturbation with respect to the nominal continuous-time system without sampling effects. Starting out with a time-delay model, in [87], it demonstrated how small-gain arguments can be used for the MSI analysis for linear systems. Moreover, a less conservative model, which uses the sampling-induced error instead of a time delay to represent sampling effects, is leveraged to reduce the conservatism of the MSI analysis. Further improvements can be obtained by considering IQCs instead of small-gain arguments [42]. The input-output approach is extended to nonlinear systems using small-gain arguments in [24] and using IQCs and exponential dissipativity in [95]. Moreover, an extension to discrete-time systems is proposed in [MH20][1].

Another framework often employed in the literature is based on hybrid system models. In this framework, the system with aperiodic sampling is modeled as an impulsive/hybrid system with continuous and discrete dynamics and this model is leveraged in the stability analysis. In [92], such a model is used together with small-gain arguments to derive an MSI bound. In [14], [19], [51], [91], [93], MSI bounds are derived using Lyapunov functions, which leverage the hybrid system model by allowing discontinuities at jumps of the hybrid system. The bound from [19], [93] is significantly improved in [52], [MH2]. It should be noted that this framework is often considered in the context of NCS with transmission

[1]Throughout this thesis, publications (co-)authored by the author of this thesis are marked with MH and listed separately in the bibliography.

protocols. In many of the previously mentioned references [19], [51], [52], [MH2], [92], multi sensor setups with transmission protocols are also considered. In such setups, the maximum allowable transmission interval (MATI) is studied as a generalization of the MSI with the MSI problem as a special case.

Besides these main frameworks, there is also a number of different approaches like using exact discretizations with embeddings [41], [59] or approximate discretizations [141]. For the MSI problem for discrete-time systems, also switched systems are considered [MH20]. More detailed overviews over approaches for the MSI analysis can be found in [58], [145]. It is important to note that the best approach may depend on the specific system dynamics that are considered. It is in general not a priori clear which approach leads in a specific setup to the best MSI bound.

In addition to research toward less conservative approaches to derive MSI bounds, some work aims also at achieving larger MSI bounds by considering additional information. Additional knowledge of minimum sampling intervals is, e.g., used in [14], [53], [72] in the MSI analysis. A bound on the average sampling interval is used in [54] to significantly increase the MATI. In [MH14] a window based constraint on possible sampling sequences is used to increase the MATI. An approach for the analysis of aperiodic sampling tailored to multiprocessor systems is proposed in [78].

1.1.2 Event-based sampling

In this subsection, we give an overview of the literature on event-based sampling. We focus on results that are relevant in view of the contributions presented in this thesis. Although there had been earlier research on event-based control, e.g., in [11], [25], [45], [88], the findings from [8], [9] provided an important starting point for current research. In [8], it is shown in a simulation that a simple threshold-based triggering rule can reduce the CPU load required for control significantly with only minor degradation of the control performance compared to periodic sampling with fixed constant sampling period. In [9], it is proven for first-order stochastic systems that using a threshold-based triggering rule outperforms periodic sampling with a fixed sampling period at the same average sampling rate in terms of the resulting variance. Both papers thus highlight some of the potential advantages of event-based sampling and thus motivated the development of systematic design approaches for event-based control mechanisms. In [142], an approach for the ETC of distributed linear systems is proposed and stability guarantees as well as a limit on the performance degradation are derived, thus theoretically confirming the results from [8]. In [50], an ETC approach based on a set-based triggering rule is presented including a thorough theoretical analysis. Of particular relevance for this thesis is [120], where a relative threshold triggering rule based on input-to-state stability for general nonlinear systems is proposed, which forms a basis for many other triggering mechanisms. The triggering rule in [120] compares the magnitude of the sampling-induced error to the magnitude of the system state and triggers when a threshold is exceeded. The controller is emulated, i.e., it is designed for the closed-loop system with continuous feedback such that an input-to-state stability (ISS) property with respect to the sampling-induced error holds. Guarantees for asymptotic stability of the resulting closed-loop system and a lower bound on the sampling intervals are derived. This bound also excludes Zeno behavior, i.e., the occurrence of an infinite number of sampling instants in a finite time interval, cf. [61].

Following [120], various papers have considered different setups and ETC approaches and provided theoretical analyses. Approaches to derive relative threshold triggering rules have, e.g, also been presented in [143] based on passivity, in [73] using input-to-state stability and a small-gain condition, in [1] based on a hybrid clock and Lyapunov arguments similar to those from the MSI/MATI results [19], [93], in [12] based on Riccati equations and in [119] using IQCs. In [76], an ETC approach that bounds the deviation between the trajectories of the closed-loop system relative to the simulated trajectories of the closed-loop system with continuous feedback is proposed. Instead of a relative threshold that depends on the system state, an absolute threshold is used in the triggering rule of [76]. Both relative and absolute threshold approaches can also be combined to a mixed threshold approach [33]. An approach with an exponentially decreasing threshold is proposed in [116]. A Lyapunov-function based triggering rule, which triggers as soon as the Lyapunov function violates a bound that depends on its value from the last sampling instant, is introduced in [133]. ETC with a triggering rule based on reachable sets is studied in [16]. The concept of dynamic ETC, where the triggering rules possesses internal dynamics, was proposed in [43], [90]. In [43], a filtered version of a threshold-based triggering rule is used. Conceptually similar, an integrated version of a threshold-based triggering rule is used in [90]. Dynamic triggering conditions are also, e.g., used in [12], [31].

ETC approaches have also been adopted to more involved setups that occur in NCS. For example, in [31], [83], [85], [135], ETC approaches with local triggering rules for decentralized systems are studied. ETC approaches that use system outputs instead of the whole state to determine sampling instants are presented, e.g., in [1], [31], [33], [66], [121]. Delays are, e.g., considered in [12], [143]. There are also many ETC approaches for multi-agent systems, see, e.g., [29], [32], [116].

Using ETC to save resources in the context of model predictive control (MPC) and to leverage benefits of MPC is, e.g., studied in [15], [46], [68]. The concept of rollout ETC, where triggering times and control inputs are jointly optimized in a receding horizon fashion is examined in [6], [137], [138]. Further, event-triggered state estimation, where a remote observer obtains state information only sporadically at sampling instants that are determined by a triggering rule is, e.g., discussed in [69], [99], [110], [126].

The analysis of minimum sampling intervals for ETC has also received considerable attention in the literature. It is pointed out in [13] that disturbances can invalidate lower bounds for sampling intervals, which are typically derived in the literature for the disturbance free case. In particular, a disturbance sequence is constructed in [13] that can lead to an infinite number of sampling events in finite time. To avoid this, time- and space regularization are proposed in [13]. Time regularization ensures that after a sampling instant, the ETC mechanism sleeps for a guaranteed minimum sampling interval, after which the triggering rule is used again to determine sampling instants. Space-regularization adds a small constant to relative threshold triggering rules. This can also yield robustness to measurement noise [112]. As an alternative, event-holding control has been proposed, where samples are only triggered after the triggering rule was satisfied for a given minimum time [111], [131].

In [100], [106], more detailed analyses of the inter-event times are conducted. For triggering rules that lead to similar trajectories as with continuous feedback, it is shown in [100] that the sampling behavior is determined by the eigenvalues of the system. An analysis based on fix points of an angle mapping is proposed in [106]. Both approaches are however limited to two dimensional systems. As an alternative, tools for the numeric analysis of inter-event times are presented in [2], [3], [117].

There are also various approaches to improve the understanding of ETC through developing unified approaches for the analysis and design of triggering rules. A framework based on hybrid systems and Lyapunov functions is proposed in [101]. Besides unifying various triggering rules from the literature, it is demonstrated in [101] that the framework also allows to design novel triggering rules. A more general version of the framework is proposed in [21]. A framework based on a hybrid small-gain theorem is presented in [77]. The framework not only allows to analyze the stability of various triggering rules from the literature, but also leads to redesigns that may improve the sampling performance for such triggering rules. In [105], a passivity-based framework for the synthesis of relative threshold triggering rules is presented.

A potential drawback of ETC is the need for continuous monitoring of the triggering rule, which typically requires dedicated hardware that is not always available. This can be relaxed by checking the triggering rule periodically, as proposed in [48] under the term periodic ETC (PETC). Note that for linear systems, PETC is equivalent to ETC for discrete-time systems [35].

Another alternative to ETC, which avoids continuous monitoring of the triggering rule, is self-triggered control. In STC, the sampling mechanism determines at each sampling instant based on sampled state information when the next sample should be taken. Thus, STC can be considered proactive, whilst ETC is reactive [49]. STC has been initially proposed in [128], where benefits of STC for task-scheduling have been demonstrated by simulation. The benefits of STC in a real-time system context were also demonstrated simulatively in [67]. In [84], [134], self-triggered control approaches for linear systems with an explicit state-based function to determine sampling intervals are proposed and stability guarantees are derived.

STC mechanisms with state predictions based on Lipschitz continuity properties are, e.g., given in [124], [125]. Both mechanisms determine a lower bound on the time when an absolute threshold ETC triggering rule would trigger. Relative threshold triggering rules in combination with Lipschitz continuity properties are, e.g., considered in [73], [103], [132]. An estimate based on a Taylor approximation is used in [28].

For linear systems, exact discretizations allow future values of the system state to be calculated precisely. This is done, e.g., in [82] where a Lyapunov function based triggering rule is evaluated exactly for different candidates for the next sampling instant. A trade-off between the computational complexity and the quality of approximation of the ETC triggering rule can be achieved. An STC strategy based on the expected evolution for a quadratic cost function is presented in [44]. For nonlinear systems, exact discretizations cannot be used. Instead, homogeneity is leveraged as an alternative in [5] to approximate a relative threshold triggering rule. This is further developed for general nonlinear systems using approximations for isochronous manifolds in [4] and based on finite-state abstractions in [26], [27]. STC based on reachable set for linear systems is considered in [16]. Results about efficient scheduling of sampling instants are presented for linear STC in [107].

Finally, we highlight the recently published thesis [136], which was completed at the University of Stuttgart. The thesis [136] contributes to the same field as this thesis. However, it has a focus on different specific problems. In [136], a data-based approach to study the MSI problem for systems with unknown dynamics is proposed. Moreover, rollout ETC, i.e., a predictive optimization-based ETC approach is studied and combined with a token bucket network model. Both contributions differ clearly from the contents of this thesis, which we will present in detail next.