Applications and developments of Barodesy

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Chapter 1

Introduction, overview and basic notation

Material models are the backbone of every modern finite element calculation. With the increase in computing capacity and the improvement of numerical methods, more and more finite element calculations are being carried out. In order to obtain realistic calculation results, advanced material models are required, especially in geotechnics.

The aim of research is to develop better and – maybe even more important – simpler material models. In this work, the material model Barodesy is improved. This recently developed material model differs fundamentally from elastoplastic constitutive models, since it requires neither yield surfaces, plastic potentials nor hardening laws. Barodesy can be written in one equation, what makes it similar to Hypoplasticity.

1.1 Overview

This thesis summarises my research on and with constitutive models, which I have undertaken in the last four years at the University of Innsbruck during my PhD studies.

The second chapter consists of the paper "Zur Rolle der Materialmodelle beim Standsicherheitsnachweis" from Kolymbas *et al.* [65] in geotechnik. It is mainly concerned with the fundamental question of whether the material model is important for the determination of stability or not. Statements of the Working Group for Numerics in Geotechnics are discussed in detail and compared with calculation results from the literature.

The third chapter takes a closer look to the basic behaviour of soil and presents the material model Barodesy. The ability of Barodesy to depict soil behaviour is explained in more detail. At the end of this chapter, the two current versions of Barodesy for clay and sand are presented. This chapter is based on the publication "Konzepte der Barodesie" published by Medicus *et al.* [78] in Bautechnik.

In the fourth chapter, improvements of Barodesy for sand are developed. On the one hand, the existing formulation of a kernel function will be improved and simplified, and on the other hand, another critical state line will be introduced, which requires to change some scalar functions of Barodesy.

In the fifth chapter, advanced stress paths (principal stress rotation) with different material models (different elastoplastic, hypoplastic and barodetic material models) are calculated and the results of the calculations compared with results of laboratory tests. These results were also included in the paper "Deformations induced by principal stress rotation modelled with different constitutive relations" (submitted to the International Journal for Numerical and Analytical Methods in Geomechanics without the improved Barodesy version and without a detailed presentation of the material models).

The sixth chapter deals with the stability of infinite slopes. Different approaches for calculating the stability of infinite slopes are presented. Various material models are used for the calculation of failure in simple shear simulations. It is possible to derive simple formulas for certain special cases. A large part of this work is published in "Stability of infinite slopes investigated with Elastoplasticity and Hypoplasticity" by Schranz and Fellin [106] in geotechnik. The here presented work is extended by calculations with Barodesy, which were not included in the original.

Beside of the work on constitutive models there has been also further research on the implementation of constitutive models into Finite Element software Chen *et al.* [15] and the estimation of material parameters under the aspect of the safety concept in the new standardisation Schneider-Muntau *et al.* [104].

1.2 Basic notation

1.2.1 Stress, strain and stretching

In this thesis the effective Cauchy stress Tensor is denoted as $\boldsymbol{T}$, following the sign convention of the continuum mechanics (tension positive). In the case of principal normal stresses, the components of the stress tensor are written with a single index, for the smallest stress component (the largest absolute value) carries the index 1, the largest stress component (the smallest absolute value) the index 3 ($T_1 \leq T_2 \leq T_3$). If a general stress state is described, two indices are used: T_{ij}. In the case of $i = j$, these are normal stresses and for $i \neq j$ these are shear stresses.

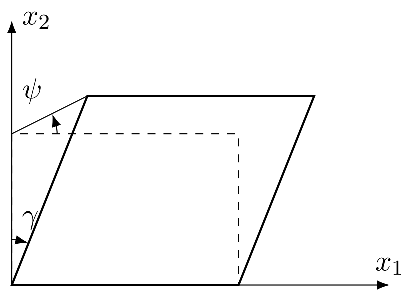


Figure 1.1: Deformation in a simple shear test

The strain $\boldsymbol{\varepsilon}$ also follows the sign convention of continuum mechanics (elongation positive). All constitutive models used in this thesis are of the rate type. Hence, in the following the rate of deformation tensor (also called stretching) $\boldsymbol{D}$ is used, which is the symmetrical part of the velocity gradient. In the case of rectilinear deformations and the use of the logarithmic strain, $\boldsymbol{D} = \dot{\boldsymbol{\varepsilon}}$ applies (Gurtin and Spear [49]).

1.2.2 Dilatancy

Dilatancy is something very characteristic for granular media like soil and was first scientifically described by Reynolds [95], although the effect was already known earlier.

In the case of granular media, dilatancy is the change of volume during a shear deformation. Different dilatancy measures are common in soil mechanics, they are partly closely related to the constitutive model which is used. Due to the importance of dilatancy and its frequent occurrence in this work, as well as its different definitions, it should be discussed in more detail in this section.

The simplest case, where the dilatancy can be seen, is in a plain strain simple shear test (cf. Fig. 1.1). In this test the horizontal normal strains are constraint and only vertical normal strain and shear strain can occur. The dilatancy angle ψ is then defined as

$$\tan\psi = \frac{\mathrm{d}x_2}{\mathrm{d}x_1} = \frac{D_{22}}{2D_{12}} \quad . \tag{1.1}$$

In elastoplastic formulations dilatancy is related to plastic flow. Therefore the plastic stretching $\boldsymbol{D}^{\mathrm{p}}$ is used with

$$\boldsymbol{D} = \boldsymbol{D}^{\mathrm{e}} + \boldsymbol{D}^{\mathrm{p}} \tag{1.2}$$

following the decomposition of strain in an elastic and plastic part. That yields

$$\tan\psi = \frac{D^{\mathrm{p}}_{22}}{2D^{\mathrm{p}}_{12}} \quad . \tag{1.3}$$

For general deformations, the equation for the dilatancy angle for an elasto-plastic model is defined as

$$\sin\psi = \frac{\operatorname{tr}\boldsymbol{D}^{\mathrm{p}}}{|D^{\mathrm{p}}_1 - D^{\mathrm{p}}_3|} \quad . \tag{1.4}$$

Here the maximum and the minimum principal stretching are used. For the triaxial test this equation results in

$$\sin\psi = \frac{\operatorname{tr}\boldsymbol{D}^{\mathrm{p}}}{-2D^{\mathrm{p}}_1 + \operatorname{tr}\boldsymbol{D}^{\mathrm{p}}} \quad , \tag{1.5}$$

with the axial plastic stretching D^{p}_1. The definition of the dilatancy as a function of plastic deformations is problematic, since plastic strains cannot be measured during a laboratory test (before unloading) and it is not applicable for constitutive relations without plastic strain (e.g. Hypoplasticity or Barodesy). This problem is sometimes overcome with the assumption that the entire deformation is plastic and hence $\boldsymbol{D} = \boldsymbol{D}^{\mathrm{p}}$. This assumption holds true only in the critical state.

Chu and Lo [16] use a different measurement for the dilatancy. They use $\tan\beta$ which is just defined for axisymmetric states and reads

$$\tan\beta = \frac{-\operatorname{tr}\boldsymbol{D}}{D_1} \quad . \tag{1.6}$$

A further possible measure for dilatancy, which is more general, is

$$\delta = \frac{\operatorname{tr}\boldsymbol{D}}{\|\boldsymbol{D}\|} \quad , \tag{1.7}$$

where the trace of the stretching $\boldsymbol{D}$ and its absolute value $\|\boldsymbol{D}\| = \sqrt{\operatorname{tr}\boldsymbol{D}^2}$ are used. The range of δ is between $-\sqrt{3}$ for hydrostatic compression and $\sqrt{3}$ for hydrostatic extension. The relation between $\tan\beta$ and δ can be calculated for axisymmetric states as

$$\delta = \frac{\tan\beta}{\sqrt{1 + \frac{(1+\tan\beta)^2}{2}}} \quad . \tag{1.8}$$

This relation is shown in Fig. 1.2

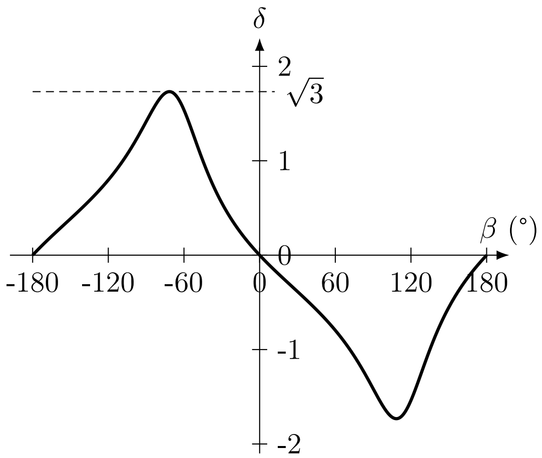


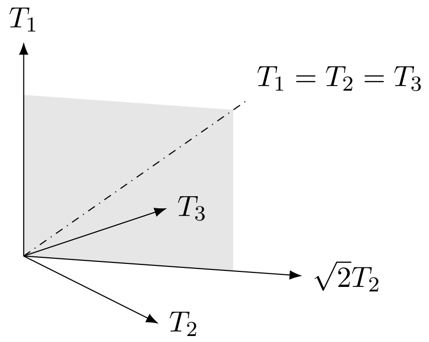
Figure 1.2: Relation between the two dilatancy measurements $\tan\beta$ and δ

1.2.3 Rendulic plane

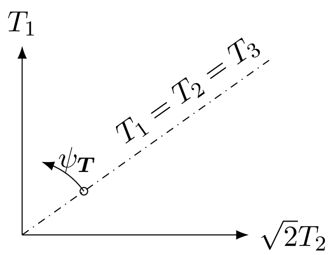
The Rendulic plane is a plane in the principal stress/strain space, which was introduced by Rendulic [94]. This plane includes the hydrostatic stress/strain axis ($T_1 = T_2 = T_3$ or $D_1 = D_2 = D_3$), Fig. 1.3a. This plane has the advantage that axisymmetric stress or strain paths (for which is $T_2 = T_3$ or $\varepsilon_2 = \varepsilon_3$) are shown undistorted, for this reason the value on the horizontal axis (T_2 or D_2) is scaled by the factor $\sqrt{2}$ (cf. Fig. 1.3). Following Gudehus and Mašín [47], in this plane the angles ψ_T and ψ_D can be defined. For isotropic compression ψ_T and ψ_D are defined to be zero, so the angles are

$$\psi_T = \arctan\frac{T_1}{\sqrt{2}T_2} - \arctan\frac{1}{\sqrt{2}} \quad , \tag{1.9}$$

$$\psi_D = \arctan\frac{D_1}{\sqrt{2}D_2} - \arctan\frac{1}{\sqrt{2}} \quad . \tag{1.10}$$



(a) Rendulic plane in the principal stress space (gray)



(b) Definition of ψ in the Rendulic plane

Figure 1.3: Rendulic plane in the stress space (it is also valid for strains)

Chapter 2

The role of constitutive models

The stability of a geotechnical building is often assumed to be not very sensitive to the choice of the constitutive model according to the Empfehlungen des Arbeitskreises für Numerik in der Geotechnik – EANG [35] from the German Geotechnical Society section 2.6.3.2 "Stoffmodelle für Standsicherheitsberechnung". However, this statement can mislead to wrong conclusions or incorrect use of soil models, if an engineer is not experienced in this field. This chapter investigate possible limits of this statement. Commonly it is emphasised, that nowadays the constitutive models plays an important role, since the computing capacity and the numeric methods are sophisticated. The EANG also mentions, that the typical elastoplastic model with a Mohr-Coulomb yield-surface (in the following MC-Model) is sufficient. Together with the information, that for excavations and slop stability analysis the dilatancy is negligible, because of the small restraint of the deformation[1] [35, p. 60] (see also Davis [21]), this can mislead someone to use the MC-Model unconsidered.

A major reason for the great acceptance of such oversimplified statements may be, that there is a long tradition of a dichotomy between stability analysis and the deformation problems in geotechnics. In the beginning just stability analysis was considered in geotechnical engineering (using a somehow arbitrary defined level of safety), relatively late also deformation problems were encountered. Always with the knowledge, that the accuracy of the deformation prediction is quite lower than the one of the stability calculation. This dichotomy is also still consistently implemented in teaching.

At the beginning the research focused on the event of the "failure" or "fracture" and tried to describe it with various strength hypotheses. For a long time deformations have been calculated using elasticity theory. With the introduction of the Elastoplasticity theory one began to realise, that failure is nothing different than a special form of a deformation state. Furthermore, Elastoplasticity theories insinuate that failure occurs precisely when a certain stress state is reached. It is ignored that the failure of solids is rather a process

[1] The original Phrase in [35] reads: Für Baugruben und Böschungen zeigt sich auf Grund der relativ geringen Behinderung der Verformungen nur ein vernachlässigbarer Einfluss des Dilatanzwinkels auf die Standsicherheit,. . .

than a state, which is difficult to determine, as it is also associated with the breakdown of numerical methods and the loss of controllability of experimental methods. An expression of "trivialisation" of failure is the term of the equilibrium equation, which found its way into the standardisation. The trivialisation consists in the belief that failure can be described by an "algebraic equation". This may be possible for states, but not for processes. By the way, the fact that failure is more a process than a singular event was already taken into account by Terzaghi and Peck [113] with the introduction of the concept of the progressive breakage, which was hardly followed up.

So called *strength reduction methods* are used in finite element calculations. The values of the used friction angle φ and the cohesion c are reduced until a limit state is reached in the case of φ-c-reduction. This results in localisation of deformation in a thin area, which can be compared with "traditional" sliding circles. Compared to kinematic methods (like sliding circles or rigid-body failure mechanisms), the finite element methods offers a quite good fulfilment of the global equations of forces and momentum. However, it must be emphasised that the application of strength reduction methods do not always deliver unambiguous results. The safety obtained with such methods depends strongly on the concrete application of the reduction and on the individual material parameters, (c.f. Fellin [29] and Zhao *et al.* [132]). In any case, the φ-c-reduction shows that the aforementioned dichotomy between deformation calculation and stability analysis fails. With a constitutive model that is unsuitable for the problem, an incorrect stress path to the limit state is obtained and therefore also inaccurate stress states at the failure are obtained. This applies in particular to undrained conditions and was one of the reasons for the overestimation of the shear strength at the accident next to the Nicoll Highway (see section 2.3.2). Moreover, the φ-c-reduction can only be used in models, which explicitly contain these parameters, such as the Mohr-Coulomb model. For the case that other constitutive models are used different methods have to be applied. For example, Schneider-Muntau *et al.* [103] reduce the critical state friction angle φ_c and the specific volume parameter of the critical state line N.

2.1 About the Failure

In the following, failure should be limited to failure of soil samples in laboratory tests, especially to triaxial tests. From a phenomenological point of view, failure manifest itself in a horizontal tangent of the stress-strain curve (a so-called limit state). What is quite often missed is that neither stresses nor strains can be measured directly, it is only possible to measure displacements and forces. In order to deduce stresses and strains from these measurements, one needs a substantial assumption that the deformation of the specimen is homogeneous, i.e. that it retains its cylindrical shape. However, experience has shown that near the limit state the deformation get inhomogeneous, the specimen is deformed unevenly, it bugles out or it is pervaded by shear bands. For a while it was believed that the loss of homogeneity was caused by disturbing boundary effects. It was therefore attempted to eliminate the friction at the end of the sample by means of lubrication and/or to make the sample more compact, but it was found that inhomogenization could not be avoided. Later, the theory has shown that internal inhomogenization is unavoidable (see section 2.1.2), because at some point of the experiment the so-called controllability gets lost, i.e. it is not possible to force the distribution of stresses and deformations *within* the sample by applying stresses and displacements to the boundary of the sample. If the controllability is lost, the sample – loosely formulated – can decide for itself which deformation it will undergo. The specimen often "choose" a localised deformation that takes place within thin shear bands. The appearance of shear bands is a kind of pattern formation in an originally uniform sample. The possibility of inhomogeneous sample deformation as an alternative to the homogeneous deformation which has been introduced so far is accompanied by the loss of uniqueness of the solution of the underlying problem of the initial boundary value problem and by a bifurcation of the solution path, whereby the solution found thereafter, e.g. in a finite element calculation, is mesh dependant. So it can be seen that the closer one get to the limit state, the less informative the experiments become. The bifurcation of the deformation manifests itself in the stress-strain-curve in the sense, that it cannot be traced to the peak by a laboratory test, as it is already distorted by the occurrence of the inhomogeneous deformation.

Failure can be seen as a kind of phase transition that begins at a single nucleus (such as a small imperfection) and transform the material from a solid to a material that is able to flow (but only in one direction). This concept has already been adopted in fracture mechanics and also puts a basic assumption of our standard numerical simulations in question. This is the assumption of the simple material, which states that the size of a specimen does not play a role in its stress-strain-behaviour. In fact it is observed that larger samples have a lower strength.

Deformations, which can be small or large, are closely linked to failure. Whether these deformations occur abruptly or slowly, i.e. if the material behaves ductile or brittle, is another question.